High-Press Questioning and Student Discourse

in a Fifth-Grade Mathematics Classroom

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Abstract

This research project examines the effects of student discourse and teacher questioning on students' conceptual understanding in a fifth-grade mathematics classroom. The research examines these two pedagogies from both student and teacher perspectives. The primary focus is on student learning: How does student discourse and high-press questioning affect students' conceptual understanding of mathematics? I collected the majority of the data during a five week period at the start of a school year in two heterogeneous fifth-grade classes taught by the researcher. Using mixed methods analysis I found that these pedagogies promoted on-task student talk enabling students to develop and refine their understanding of the mathematics. The pedagogies also promoted cooperation within the classroom as students worked together to conceptualize some of the big ideas they investigated. The secondary question addressed the teacher's experience implementing these student-centered pedagogies. In this respect, student discourse served as a highly valuable formative assessment, while high-press questioning facilitated student thinking.

Keywords: high-press questioning, student discourse, conceptual understanding, teacher-researcher

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Chapter 1: Literature Review and Rationale for the Study

The Numbers Don't Add Up

The State of Washington administers a yearly set of exams to students to test proficiency in Reading, Writing, Science and Mathematics. It is by no means a perfect measure of learning or comprehension, but these criterion-referenced tests do provide us with a picture of student achievement. The percentages of students who pass the mathematics exam at a "proficient" level are those who have demonstrated an understanding of Washington State Standards for the given grade (Table 1.1). These percentages indicate that there is a problem with mathematics instruction and student learning. Student achievement is not where we want it to be. This problem becomes amplified as the scores are interpreted to define student aptitude. Those who are not proficient for one reason or another are not considered productive mathematicians; this can then become part of a child's identity. In 2009 the percentage of students who graduated from college with a degree in mathematics was one-third of what it was in 1970. Since the late 1990s is has hovered around 1% (National Center for Education Statistics, 2011). This is despite the increased focus on mathematics in the K-12 system. Students who avoid mathematics find themselves less equipped to function in a world that requires increasing

Table 1.1. Percent Proficient in Mathematics

Grade	2010-11	2009-10	2008-09	2007-08
3rd	61.6	61.8	66.3	68.6
4th	59.3	53.7	52.3	53.6
5th	61.3	53.6	61.9	61.2
6th	58.8	51.9	50.9	49.1
7th	57.0	55.3	51.8	50.5
8th	50.4	51.6	50.8	51.8
10th	NA	41.7	45.4	49.6

MSP/WASL State Standardized Test
(OSPI Report Card, 2011)

mathematical reasoning (Boaler & Greeno, 2000; Checkley, 2001). The purpose to which we put mathematical knowledge today is not the same as it was one hundred years ago, when American schooling followed the trends of the industrial age and assembly line mentalities. Robert Moses, civil rights activist and educator, makes the point that "People who don't understand algebra today are like those people who couldn't read or write in the industrial age. Computers have made elementary mathematics as important as reading and writing (Checkley, 2001, p. 6)." Today, students need to leave high school with a deeper understanding of mathematics than simply procedural knowledge.

A closer examination of the state testing data reveals an even greater problem facing schools across the nation: inequity. Which of our students are meeting standard? Which are not? Are the results predictable? The term “achievement gap” is a commonly used phrase that might bring to mind a set of divergent lines on a graph representing growth patterns for different groups of students. Table 1.2 shows trends in grouped math scores that appear somewhat disturbing.

Table 1.2. Percent Proficient in Math by Group

Group	2010-11	2009-10	2008-09	2007-08	2006-07	Average
American Indian	34.9	34.4	40.9	42.4	39.7	38.46
Asian	76.8	70.8	73.6	73.4	n/a	73.65
Black	37.2	34.1	42.9	39.5	38.1	38.36
Hispanic	40.9	33.7	39.1	39.6	36.6	37.98
White	65.9	59.6	69.2	67.8	66	65.7
Limited English	22.8	16.8	17.7	20.5	12.9	18.14

MSP/WASL State Standardized Test
(OSPI Report Card, 2011)

These results have consistently been disparate and predictable, which indicates a pattern of inequity; the gap is a more complicated issue, though, than what test scores alone illustrate.

The achievement gap is a measure of inequity in our school systems and our culture at large (Gutiérrez, 2002). Social conventions of privilege, sometimes hidden or subtle, have served over time to provide an easier path for some groups at the expense of others (Johnson, 2006). Educator and researcher Gloria Ladson-Billings (2006) makes a clear and compelling case that our nation's continued yearly deficit in the equity of education has amounted to a national debt. Like the nation's financial debt, she says this "moral debt" must be addressed systematically and in a timely manner. Working against this tide of privilege is the domain of *equity pedagogy*. Equity pedagogy has as its central goal the elimination of patterns of success based on student characteristics (Gutiérrez, 2002). In the end, equity is about erasing unearned power relationships. Addressing unearned privilege in our culture must begin with equity in education, and because mathematical literacy is so central to full participation in our society, it is critical to confront patterns of inequity in math (Checkley, 2001; Gutstein, 2003; Johnson, 2006).

Reform-oriented curricula were designed to help create a more equitable, interconnected, and meaningful mathematics experience for students. In 2000, the National Council of Teachers of Mathematics (NCTM) published *Principles and Standards for School Mathematics,* which was a follow up on their previous effort in 1989 (Van de Walle, 2010). *Principles and Standards for School*

Mathematics lays out a new vision for the landscape of math education beginning with six principles: Equity, Curriculum, Teaching, Learning, Assessment, and Technology. Equity was not an afterthought: high expectations and support for all students is a central component of the NCTM framework. Students and teachers work together, actively engaged in problem solving activities that push students to develop strong reasoning skills (Van de Walle, 2010). This is a significant departure from the traditional approach of lecture, seatwork, memorization, and rote practice. As a result of these new ideas, a great deal of work has taken place to transform math education.

Pedagogies that Support Equity in Student Learning

Classroom discourse. Though there has been progress in mathematics education over the last twenty years, even now low level skills are taught as the predominate way of doing mathematics in most classrooms (Ball, 2001). One of the challenges is that teaching mathematics is an inherently complex task. Core activities of mathematical instruction include: "figuring out what students know; choosing and managing representations of mathematical ideas; appraising, selecting, and modifying textbooks; deciding among alternative courses of action; [and] steering a productive discussion" (Ball & Bass, 2000, p. 88). Teaching is not a generic practice: it requires constant instantaneous reflection upon a wide array of pedagogical and mathematical topics (Ball & Bass, 2000). One of the ways teachers can develop an atmosphere that nurtures conceptual understanding is to attend to the level of discourse in the classroom. Research shows that focusing on understanding before skill development results in a deeper conception of the mathematics that help students *understand* the procedures rather than just *use* them (Boaler & Greeno, 2000; Philipp, 2000; Wood, 1998).

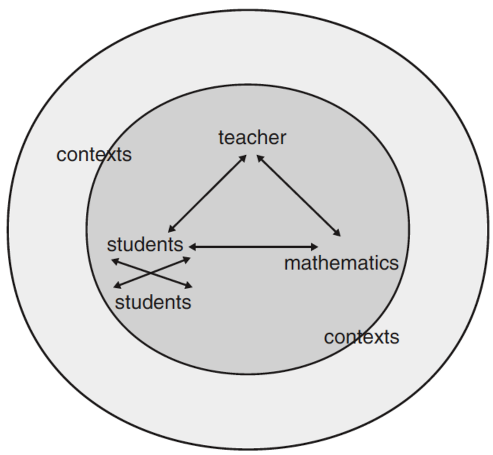
There is a wide range of topics that fall under the umbrella of classroom discourse. Discourse involves teacher beliefs and pedagogies, the practice of questioning, student identity and disposition, justification, classroom norms, and a vision for equity. In Western philosophy the idea that discourse through questioning could be used as a tool to promote understanding goes back to Socrates, though this is by no means an exclusively western belief. High-press questioning is different than the Socratic dialog in that the teacher does not frame knowledge within the question itself. When Socrates questions Meno's slave (Appendix A), he asks him eleven questions: eight of which are answered in the affirmative, and in three the slave gives sought-after numerical responses (Plato). Here Socrates inquires about the area of a square, but the answers are embedded in the questions themselves:

> **Socrates.** And if one side of the figure be of two feet, and the other side be of two feet, how much will the whole be? Let me explain: if in one direction the space was of two feet, and in other direction of one foot, the whole would be of two feet taken once?
> **Boy.** Yes.
> **Socrates.** But since this side is also of two feet, there are twice two feet?
> **Boy.** There are.
> **Socrates.** Then the square is of twice two feet?
> **Boy.** Yes.

High-press questioning, on the other hand, seeks to elicit reflective thinking, analysis, and evaluation rather than agreement or understanding (Loska, 1998).

Questions like, "How do you know?" are completely open-ended and ask the student to make the connections that Socrates himself makes within his questions. Social constructivism supports the idea of high-press discourse within a student-centered classroom. Based on theories of learning by Piaget and Vygotsky, social constructivism views social interaction as fundamental to learning.

Figure 1. A Model of Classroom Discourse



(National Research Council, 2001)

Discourse within a student-centered classroom can be anything from whole class discussions to small groups of students working autonomously. For me, the diagram illustrates two-way communication, access to mathematical resources, and teacher facilitation. All of this exists within a larger context, which might be defined by the task, or more generally by the community of practice.

Classroom discourse from the social constructivist perspective is highly valued and it can be manifested in many ways (Figure 1). The challenge is that regularly implementing productive classroom discourse that meets both the needs of the students and addresses the school curriculum is not an easy task. Fortunately there is a significant body of research that has examined the topic (Ball, 2001; Boaler & Staples, 2008; Bransford, Brown, & Cocking, 2000; Wood, 1999).

Constructivist theorists believe that due to the socially constructed realities of diverse learners, there will be correspondingly diverse pathways to learning, so teachers need to consider how to provide access for all students to enter the discussion. Khisty and Chval (2002) conducted two case studies of elementary teachers during mathematics instruction. The contrast they drew focused on the language the teachers used and how it enabled students to discuss and make meaning from mathematics. When one of the teachers in their study scaffolded the use of specialized mathematics vocabulary and modeled appropriate justification, her students appropriated these into their own ways of being in the classroom. Her teacher talk served to extend and draw out student thinking rather than just fill in gaps where students seemed to have misconceptions (Khisty & Chval, 2002).

Sometimes students can offer explanations of their work that seem correct on the surface, but upon further examination reveal misconceptions or insufficiently developed understandings (Donnovan & Bransford, 2005). For example, using addition to answer a multiplication problem would yield a correct answer, but this strategy might not be efficient and it could hide gaps in student understanding (Cobb, Yackel, & Wood, 1992). Agreements as to what constitutes proof of mathematical argumentation help students internalize the need to understand concepts more deeply. The public expression of their ideas allows students to communicate with one another about their conceptions. As students learn how to participate in this discourse, they become more aware of their own and their classmates' patterns of thought (Boaler & Staples, 2008; Wood, 1999). Research has indicated that student comprehension, productive dispositions, and achievement can all be elevated through a sustained focus on high quality classroom discourse (Boaler & Greeno, 2000; Kazemi & Stipek, 2001; Khisty & Chval, 2002; Wood, 1998).

Sociomathematical norms. Yackel and Cobb (1996) coined the term *sociomathematical norms* to distinguish a set of specialized behaviors, attitudes, and values in mathematics instruction that help define a classroom's culture. The term describes the common understandings within the *community of practice* of what counts as mathematically different, what is sophisticated, what is efficient or elegant, and what qualifies as a mathematical justification. Kazemi and Stipek (2001) point out that many of the social norms used in other classrooms have additional requirements and specialized goals in the mathematics classroom. Sharing work is a social norm, but trying to figure out the strategies other students used and attaching meaning to those strategies is a sociomathematical norm. Coming to consensus is a social norm, but when it is done through logical argumentation it is a sociomathematical norm. The negotiation of these norms is a process that develops organically over time with focused support from the teacher. Kazemi and Stipek's findings suggest that it is important to teach students how they are expected to operate within the community of practice, and continually model the normative ways of thinking in all areas of classroom discourse. These ways of being do much more than teach students how to answer the teacher's or other students' questions: they establish criteria for a more formal membership in the larger world of mathematics. Cobb and Yackel (1996) assert that the students' beliefs and the sociomathematical norms, which they come to internalize, continuously effect change on one another. As students respond to requests for different explanations and develop meaningful interpretations of mathematics, they interactively learn what counts. In this way, efficacious mathematical dispositions evolve: students develop a belief that everyone is capable of thinking mathematically. Lave (1996) describes the process of identity development in terms of this kind of social practice. She asserts that "who you are becoming shapes crucially and fundamentally what you 'know'" (Lave, 1996, p. 157). Sociomathematical norms can work to transform student identities. It is not just about the creation of an active, dynamic learning environment, but more importantly about the development of a community of practice.

Questioning. Teacher questioning plays a significant role in the development of student understanding in a math classroom. Questions can drive the thinking of a class and promote deeper thinking around mathematical concepts. Asking open-ended questions and providing realistic think-time allows students to process their ideas and make connections (Black, Harrison, Lee, Marshall, & Wiliam, 2004). All new learning begins with prior knowledge, and accessing these related schemas takes time. Asking students to generalize and justify their thinking helps students make meaning and find connections among a diverse range of topics (Bransford et al., 2000). Huffered-Akles, Fuson, & Sherin (2004) describe a shift from asking questions to find answers, to seeking out student thinking around mathematical concepts. When teachers understand where students are in their thinking they are then in a position to teach directly to the schema the student has created. They provide a scale of teacher talk to elucidate their framework (Table 1.3).

Table 1.3. Levels and Components of a Math-Talk Learning Community

Level	Description
0	Traditional teacher directed classroom with brief answer responses from children
1	Teacher beginning to pursue student mathematical thinking. Teacher plays central role in the math talk community.
2	Teacher modeling and helping students build new roles. Some co-teaching and co-learning begins as student to student talk increases. Teacher physically begins to move to side or back of the room.
3	Teacher as co-teacher and co-learner. Teacher monitors all that occurs, still fully engaged. Teacher is ready to assist, but now in a more peripheral and monitoring role (coach and assister).

(Huffered-Ackles et al., 2004)

The simple pedagogical move, though, of "questioning students" is not enough; the way in which teachers question is a critical element in need of examination. Herbel-Eisenmann and Brefogle (2005), and Wood (1998) delineate two fundamentally different types of teacher questions: *funneling* and *focusing*. Funneling questions, which are low press, seek a correct answer: asking students to say what is in the teacher's mind. For example, "Who can tell me the name of a triangle with two congruent angles?" Funneling questions engage the cognitive abilities of the teacher more so than the students (Wood, 1998). Focusing questions draw attention to essential elements of the problem and allow student thinking to come to the forefront: "What are the characteristics that differentiate this triangle from other triangles?" This is akin to high-press questioning. The teacher's role as the authority is greatly reduced with focusing questions, as the expectation is no longer that the teacher will reveal what is correct. Rather, the teacher will give the control back to students so that they can engage in meaningful thought (Wood, 1998). Deeper questioning can serve to help students read between and beyond the lines (Morgan & Saxon, 2006).

Kazemi and Stipek (2001) draw a similar conclusion based on a study of four upper-elementary teachers. All four teachers ranked high in positive affect and, at least superficially, the students all seemed to be engaged and supported as learners; but a closer examination revealed a qualitative difference in their questioning techniques. Two of the teachers used a high-press questioning style that brought forth the kinds of sociomathematical norms that Yackel and Cobb (1996) found to be such powerful factors for student learning. The low-press questioning style of the other two teachers did not deeply challenge student conceptions. High-press questioning made room for longer exchanges, more think time, and established norms around the level of conception that was acceptable. Kazemi and Stipek (2001) examined teacher questioning specifically, but also draw on Yackel and Cobb (1996) and the distinctions between social and sociomathematical norms. They focused on classroom practices that pushed students to develop deeper levels of conceptual thinking.

Lubienski (2000) and Parks (2010) advise some caution when using questioning techniques in diverse classrooms, noting that some students will more readily understand the academic register used by teachers than others. Lubienski focuses on the characteristics of socioeconomic status (SES) and identity. In her study as a teacher-researcher, she found that low SES students did not persist as long in problem solving and would sometimes struggle with the contexts offered in problems. Parks identified implicit questioning and explicit questioning from both traditional and reform styles of teaching by one experienced third grade teacher. The implicit questions are more open, comparable to focusing or pressing questions. Many students had difficulty answering these types of questions and were not able to engage unless more explicit, closed, or well defined questions were asked. While researchers have addressed some of these concerns (Boaler, 2002; Gutstein, 1997; Walsh & Sattes, 2005), there is a need for teachers to be conscious of challenges students may face when pressed to interpret questions and communicate their thinking, as it may be culturally less familiar to do so.

High-level student work. Henningsen and Stein (1997) studied a wide variety of factors that shape student interaction with high-level mathematical tasks. They found five factors that appeared to be generally effective in keeping students engaged with high-level mathematics. Building on students' prior knowledge was effective 82% of the time; scaffolding 73%; providing an appropriate amount of time for the task 77%; high-level performance modeled for the students 73%; and sustained pressure for explanation and meaning was effective 77% of the time (Henningsen & Stein, 1997). This data triangulates the constructivist ideas about learning from Piaget and Vygotsky with its emphasis on prior knowledge and scaffolding. Accessing prior knowledge is also an important tenant of equity pedagogy (Boaler, 2002; Gutiérrez, 2002; Gutstein, 1997). Sustained pressure for explanation and meaning confirms the high-press work done by Kazemi and Stipek, while high-level performance modeled is reflected in the sociomathematical norms framed by Yackel and Cobb. Henningsen and Stein then examined the factors that led to the decline of high-level thinking which included teachers removing the challenging characteristics of the tasks, an emphasis on the answer or finishing a set amount of work, and that of providing too little or too much time for the task at hand. The researchers showed that without intentional direction by the teacher, there are ever-present opportunities for the level of thinking to decline. High-level tasks help bring out high-level thinking (Smith & Stein, 1998). By attending to these pedagogical factors and structuring classroom discourse around the process of inquiry and emphasizing conceptual understanding above procedural knowledge, teachers can sustain high levels of mathematical thinking (Ball, 2001; Boaler & Greeno, 2000; Henningsen & Stein, 1997; Stipek et al., 1998).

Developing a Research Question

Limitations of current research. There has been a great deal of research in elementary mathematics in the exact area I intended to study. The body of research I have assembled makes a convincing set of arguments about the strengths of constructivist teaching methods that give the student some measure of control over their learning. Research has demonstrated the power of pressing questions to elicit higher order student thinking; it has also uncovered teacher pedagogies that help sustain high-level student work in the classroom. A limitation I have found with this research is that the researchers in most cases are not elementary teachers; in other words, there is frequently a distinct disconnect between teacher and researcher roles. Acting as teacher-researcher gives me a privileged perspective on the data, and thus a powerful position from which to analyze it. I hope that the voice of the teacher comes through my research and adds to the conversation.

The stubborn ounces. As you will read in Chapter 2, the classroom where I currently teach is not particularly diverse by national standards, though I think it's fair to say that there is always more diversity than we perceive in any classroom. Furthermore, the need to recognize students' existing schemas is a basic tenant of constructivism and my classroom is no exception. My students will also benefit from learning in a climate where productive classroom discourse is the norm and where conversations address individual conceptions with the goal of meaning making in mathematics.

In past years I have recognized a tendency in my own practice to "tell" students, even though it is my belief that inquiry is the optimal method of developing productive dispositions in learners. It is so easy to think that teachers can present what they know: that they can show students fully realized and orderly mathematical conceptions, and from this students will understand. My desire to transplant my schema around a given topic into the minds of my students is simply not possible, nor upon deeper reflection is it even desired. The way that we think about the world is a very personal thing, connected to the sum of our experiences, our tastes, and our interests to say the least. My understanding of two-dimensional geometry may be well formed, but it is not suitable to be transplanted. The passion I have as a learner is one that comes from the spirit of inquiry; the answers are meaningless without a deep conception of the questions. Learning is messy, and it has to be. There are new ideas that my students are being asked to integrate into their existing schemas and that integration is a job only they can perform.

I expect my research will add another ounce to the scale on the side of inquiry teaching and learning. Johnson (2010) uses a poem of Bonaro Overstreet's titled "Stubborn Ounces" to speak to doing something that you believe in despite the full knowledge that the change it brings about will not change everything. He uses it to discuss his work on the problem of privilege, but I think it also speaks to the efforts that teachers make year in and year out, fine tuning, discarding, and seeking out new ways to reach students.

> STUBBORN OUNCES
> (*To One Who Doubts the Worth of Doing Anything*
> *If you Can't Do Everything*)
> You say the little efforts that I make
> will do no good; they will never prevail
> to tip the hovering scale
> where Justice hangs in the balance.
> I don't think
> I ever thought they would.
> But I am prejudiced beyond debate
> In favor of my right to choose which side
> shall feel the stubborn ounces of my weight.[1]

Through my research and the planning and implementation of this study, I hope to develop greater skills as a facilitator to my students' learning. I will add the "stubborn ounces of my weight" to recent research findings and develop my own pedagogy along the way. I will examine how high-press questioning and student discourse affect students' conceptual understanding of mathematics in my fifth-grade classroom.

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[1] Overstreet, B. W., 1955, p. 15.

Chapter 2: Methods

Participants and Setting

The setting was a public elementary school in a small urban community. The school served about 400 students kindergarten through fifth grade. About 86% of the students were white, and roughly 12% of the students were on a free or reduced-price meal plan. The school boundary contained the suburban neighborhood surrounding the school, which was approximately 1 square mile in size. Because it was geographically smaller than most elementary school boundaries in the district, the community was relatively tight knit. The entire district served over 9,300 students, 75% white, and 27% free or reduced meals.[2]

Morning Star Elementary[3] had a low rate of student mobility. About 90% of the students in the fifth grade were in fourth grade down the hall the previous year. From the start of the school year through the end of the study, no students left the cohort and only one was added. Morning Star was often one of the higher scoring elementary schools in the district as measured by state testing. The cohort I studied came in just under the district average in mathematics with 68.3% meeting standard at the end of 4th grade. The state average for 4th grade students in mathematics that year was 59.3%. The same group had 90% pass the reading test (10% higher than the district average, and over 20% higher than the state), and 93% pass the writing (almost 20% higher than the district average, and more than 30% higher than the state). Though we quietly celebrated these numbers, the

[2] This data comes from the Office of the Superintendent of Public Instruction's website
[3] Pseudonym

school culture did not place a high level of importance on the test and there was only minimal test preparation.

There were two fifth-grade classrooms at Morning Star, each with 32 students. Because of the large class size, I had a paraeducator in the room for the length of the school day. Fifth grade is the final year of elementary school in this district, so Morning Star followed a model of teacher specialization and student rotation in the fifth grade to help the students make an easier transition to middle school. I taught math to both classes. At the beginning of the year, homeroom students were given student numbers by last name (1-32 in my room, and 33-64 in the other). The two math sections were randomly decided by taking all of the students with an odd number in one period and all of the students with an even number in the other. Two students with IEPs were pulled out of the regular classroom, so each of my math classes had 31 students. Participation in the study was open to all students from both classes.

The fifth-grade classrooms were next door to each other and had a ten-foot soundproof accordion door that opened up for whole group activities like morning announcements, class meetings, and short periods of whole group instruction[4]. There was a conscious effort to make the entire group feel like a family. Normal instruction periods were taught in the separate rooms with the doors closed. We had math class scheduled four times a week for one hour blocks of time with an additional math assessment period on Friday, which was roughly thirty minutes.

My classroom was configured into eight table groups of four. There were opportunities for whole class instruction, independent work, partner work, and

[4] Art and Health classes are team taught with the classrooms open to each other

four person groupwork. My teaching station was off to the side of the room, and I worked from a document camera which was connected to a ceiling mounted projector. I was also able to connect my laptop computer to the projector, though I didn't often do this during math class. During whole-class conversations or direct instruction, the students or I often used the document camera, and I would occasionally make notes on the white board which runs behind the screen and extends out about four feet on both sides. Students volunteered or were sometimes asked to share work on the document camera which we then discussed as a class. For the majority of the class period while the students worked on an assigned task, the paraeducator, Jamie[5], and I moved between the table groups and checked in with students, discussing the work at hand.

Math Curricula

This was the fifth year the district used Connected Mathematics 2 (CMP2) in 5th through 8th grades. It is a reform-oriented curricula whose stated goal is to "help students develop sound mathematical habits" (Lappan et al., 2009). The mathematics is intended to be taught using an inquiry approach, and the big ideas in mathematics are woven together in the materials to highlight their connections. The text lends itself very naturally to groupwork, teacher questioning, and student discourse. Most of the state standards are addressed by the CMP2 curriculum, but our district made additional materials available to us for concepts not covered in CMP2. During the length of the study period, the students completed the "Prime Time" CMP2 unit, which addresses the concepts of factorization, prime factors, common factors, multiples, and common multiples. Students also developed

[5] All participants have been given pseudonyms

skills in multiplication and division computation. Daily work during the study was a progression through the CMP2 materials, with an occasional review of some computation algorithms.

Computation was mainly practiced at home and monitored through the use of a weekly homework assignment given on Monday and collected the following Monday. Thus the week would often begin with a quick review of double digit multiplication or long division, and then move quickly into the investigations in CMP2. Homework was given from CMP two nights a week with the expectation that students would spend up to 45 minutes, if needed, making sure to explain their thinking as they answered the questions. Each week an assessment was given on the material from Prime Time. I used assessment materials from the publisher in three cases and then created two of my own assessments.

Data collection

This was a qualitative research study in which I was the teacher-researcher. I collected the majority of the data over a five-week period beginning on the eighth day of school. I conducted a multi-level case study with two different units of analysis. The first level of analysis was at the class-level and the second was at the student-level.

Teaching two classes in this study allowed me to examine and compare the effects of questioning and discourse with different groups of students. The random nature of the class make-up was intended to offer a degree of credibility for my findings. Table groups within each class were formed randomly on a computer and rotated on a weekly or bi-weekly basis. This regular rotation allowed me to observe changes in students' interaction and to foster their individual growth. Random table assignments also eliminated teacher bias in group selection, though perhaps more importantly it yielded the power of the seating chart to chance, which both equalized the students and enabled their independence. Groupwork necessitates the development of norms which address issues of status within the groups; handling my own status was also integral to the process. One of the central goals of high-press questioning and student discourse is the decentralization of the teacher as sole authority of mathematical knowledge. The table groups must have the final say as to what they feel is correct.

I was also interested in individual development. Students who volunteered to participate in the study were eligible for the student-level case study. I selected three boys and three girls who represented a wide range of scores on the 4th grade standardized test in mathematics. I interviewed these students and closely examined data I collected on them over the course of the study.

The five week study captured data from the first full unit of the year. The

Table 2.1. Schedule for Data Collection

	Week					
Method	**1**	**2**	**3**	**4**	**5**	**12**
Student Surveys	X					X
Student Interviews		X	X	X	X	
Video Recordings	X	X	X	X	X	
Weekly Quizzes	X	X	X	X	X	
Parent Questionnaire	X					

final student surveys were collected after the second unit. My research journal was written in on a daily basis. The schedule for data collection is presented in Table 2.1.

Research journal and field notes. As the teacher-researcher I was in the unique position to examine the reasons for my pedagogical moves. I made journal entries on a daily basis reporting my reflections from each class. These entries provided first hand documentation of what happened: a record of any interesting events that transpired during the period. The journal preserved what I perceived in the moment, keeping it somewhat distinct from the findings I made through a deeper analysis of the data (Hubbard & Power, 2003). At the start of the third week of the study, Jamie joined our classroom and took her own field notes at the end of each period. Jamie's reflections not only added to mine, but provided an external viewpoint that centered on student thinking. The two sets of field notes served to triangulate what I gathered using other methods.

Student surveys. Students were asked to fill out an anonymous survey that was focused on their perceptions of what was meaningful in mathematics and what they thought math looked like in school (Appendix B). They were given this survey at the beginning of the study and then again about seven weeks after the end of the study unit. The five week collection period provided ample time to collect classroom data, but I wanted a longer window of time to affect changes in student dispositions. Thus, the second administration of the student survey occurred in week twelve. This extended time allowed me to capture the changes in student conceptions more accurately. Data from the surveys was examined to help characterize the students as learners of mathematics. The initial surveys from both classes were collected and compiled. I put both classes together to protect the identities of students, but in retrospect I believe that having separate results would have offered another perspective on the groupings without compromising anonymity. The final student survey was separated by class to examine any differences from the initial findings. Changes were examined to determine if there was a relationship between the use of teacher questioning and students' mathematical dispositions. Because student dispositions can influence and be influenced by conceptual understanding, this served as a useful qualitative measure of change during the course of the study period. The survey also informed students about their own perceptions of mathematics, which then put them in a position to make changes to their existing schemas.

Student interviews. The six students who were selected for the student-level case studies were interviewed once during the course of the five-week study. The 10-15 minute semi-structured interviews asked open-ended questions designed to get the students talking about their experiences in class, their thinking about questioning, and a self-assessment of their understanding of recent work (Appendix C). Student interviews were recorded and transcribed to allow a deeper analysis of the data. All interviews ended with an open-ended request for anything that they would like to add. Interviews allowed me to probe the progression of student thinking more deeply. Interview data compared across the six students helped confirm findings from the student surveys, and provided credibility to my teacher research journal (Hubbard & Power, 2003). The interviews were an opportunity to reflect on and analyze changes in mathematical dispositions, the use of mathematical language, and questioning.

Video recording. I made and examined video recordings of math classes once a week in each class throughout the five week study. These recordings were used to capture student questioning with the exact phrasing intact. Recordings were also used to compare my pedagogy and the class's performance as they attempted identical tasks. The video allowed me to review the progression of student thinking and teacher pedagogy in closer detail, and to become aware of aspects of the classroom that I had not witnessed firsthand. This was an especially important method of data collection for me as a teacher-researcher because a teacher's internal perceptions of what happens in the classroom can sometimes differ from what one sees in video recordings. The recordings allowed a more objective perspective on the use of questioning in my classroom. The videos were all transcribed, though only segments that seemed particularly illuminating were transcribed verbatim. The bulk of the transcription was made in conversation analysis style. The transcript notes were then coded using an open coding approach (Mertens, 2010). The codes were examined, edited, and organized as themes emerged from the data. I tallied the codes captured in each lesson, entered them into spreadsheets, and analyzed the data. These transcribed and coded classroom conversations lend credibility and confirmability to my findings (Mertens, 2010).

Weekly Quizzes. At the end of each week I gave the students a short quiz to assess their understanding of the week's material. I didn't teach math on

Fridays, so this was a great way to get feedback on the students without having to sacrifice instructional time. Two of the quizzes were scheduled assessments written by the publisher and two were created by me. The final quiz was a subset of questions that I selected from publisher materials. All of the quizzes were individual pencil and paper summative assessments. The quizzes were scored, and averages for each class and the students from the case studies were tracked from week to week.

Parent questionnaire. At the start of the school year I asked all parents of fifth-grade students to fill out a questionnaire on math related experiences, dispositions, perceptions, and challenges (Appendix D). The family's community of practice around math is an important determinant of how students form mathematical identities (Boaler & Greeno, 2000). Over two-thirds of the parents indicated they would be willing to complete a questionnaire, but of that number only one-quarter returned it. This information helped construct a general picture of the families' mathematical dispositions, but I was not able to collect enough data to make any findings. Typically, teachers have very limited information on parents' mathematical dispositions, so despite the limited number of responses this information was nonetheless insightful.

Limits of Conclusions

Teaching students to value the act of questioning and to become comfortable exploring mathematics is a complex challenge. The results of this study may not be readily transferable as there are so many variables both identified and unidentified that are out of the teacher's control (e.g. family lives, student dynamics, prior knowledge, scheduling disruptions, etc). Additionally, the time period for data collection in this study was a short window for observing changes in student dispositions. The particular area of mathematics we studied may have also affected the results of my study. Finally, there may have been unexamined aspects to my pedagogy or the curriculum itself that could have impacted student learning.

In qualitative research, the researcher is the lens through which the findings are made (Hubbard & Power, 2003). Because of this, I made significant efforts to ensure reliability. I was engaged with my students every day and I made persistent observations. I used multiple data sources to triangulate my findings, and the analysis I made was peer reviewed. In Chapter 3, I will clarify my biases for the reader and attempt to provide a rich description of the study so that it can be clearly understood.

In the analysis of my findings, I attempted to let the data do the talking. The grounded theory of data analysis directs researchers to evaluate data without a preconceived set of codes in mind (Mertens, 2010). During the first round of coding on classroom videos I created 48 different codes. This evolved into 27 student codes and 8 teacher codes. As I began to analyze and condense my coding structure I remained cognizant of this principle of grounded theory as not doing so could present a threat to the validity of my findings. I used a variety of data sources to help triangulate my findings. Each source was gathered in different contexts and in different ways, but they all contributed towards my effort to answer the same questions.

I did not intend to prove causality but merely to observe the patterns in student discourse, comprehension, participation, and questioning in the two classroom communities. Research has shown that a focus on comprehension through questioning provides a superior learning environment for students (Boaler & Greeno, 2000; Kazemi & Stipek, 2001; Wood, 1999). My objective was to take the recommendations of educational researchers and see if I could duplicate their findings.

Chapter 3: Research Findings

The Lens and the Subject

My biases as an educator align well with reform-oriented curricula. I believe that student talk around the mathematics they are learning is a good thing. I think that when people have an opportunity to use language it helps them make meaning. When students can talk about how factors relate to multiples, they show a real understanding of an important whole number concept. Understanding is what makes mathematics useful. Students should be the authorities of mathematical correctness: the teacher is not the only owner of mathematical understanding. Conceptual understanding must be taught before procedural knowledge. Teachers can use questioning to create a classroom of active learners. I believe that teachers' talk should not always be explanatory in nature; answers often serve as a stopping point for thought, but questions can drive student thinking.

During the time of this study I was in my sixth year of teaching, and had taught fifth grade in this district for all of those years. Though I had used groupwork in the past, I had never focused as deeply on student discourse as a means of developing comprehension as I did during the study period. As a student at The Evergreen State College during the period leading up to this research I did a great deal of research and reflection on the practice of teaching. Throughout my time at Evergreen I tinkered with ideas and implemented new teaching practices, but the deeply researched area of student discourse, questioning, and teacher press fundamentally reshaped the way I approached the classroom.

Any qualitative action research project places the teacher in the role of the lens through which data is collected and analyzed, but I also investigated the implementation of the pedagogies themselves. I was the subject of my investigation as much as the students were. This offered me a unique perspective as a researcher, but at times it was a dizzying enterprise. The beginning of the research period was marked by some internal questioning: what if I didn't teach it the way I intended? My self-doubt was lifted by the realization that my role in the research was that of researcher and whatever I captured would be data from which I might draw conclusions. As the first classroom video began to record, my focus went back to my role as the teacher and my question to the students: How do you know? This became the launching point for my examination of student communication, reasoning, and comprehension.

Findings and Outcomes

Student Dispositions. My immediate goal was to improve the students' conceptual understanding of mathematics, but I was also interested in positively shaping students' mathematical dispositions. Students' beliefs about the importance of math and their perceptions of their own effectiveness doing mathematics are closely linked to conceptual understanding (Van de Walle, 2010). I gave students a survey to assess how they thought about math in the first week of the study period and again seven weeks after the study unit. There were many interesting findings that presented themselves through an examination of the pre and post surveys. My first finding was that by the end of the study period many more students thought it was not important to appear smart to others. Secondly I found a dramatic swing in student thinking away from memorization and towards thinking for oneself. Finally I found that students assigned a greater value to understanding the big ideas in mathematics by the end of the study period. Taken together with some of the other significant changes, these differences point to an increased sense of cooperation (findings 1 and 2), and an increased focus on conceptual understanding (finding 3).

The initial student survey given in the first week of the study period revealed some interesting conceptions about mathematics. It showed that a majority of students thought that other subjects were more interesting than math, though 83% felt that math was interesting. Notably, 78% of the students agreed that math was mainly about memorization rather than thinking for yourself. In a related question, 45% strongly agreed and 38% agreed that math is a lot of procedures that have to be memorized. My first key insight from this data source was that an astonishing 83% of the students viewed math as procedures and facts to be memorized. When students were asked to rank aspects of math in order of importance, memorizing the facts was ranked the highest with 66% of the students ranking it first above understanding the big ideas, helping others learn, finishing lots of work, and learning to use calculators. Two-thirds of the student interviews corroborated this disposition, revealing a very high value placed on computation and memorization. Math was seen as a set of operations and numbers in which working with speed and accuracy was the goal. One student commented that "you just have to know how to do math to really do it." This is a very traditional perception of mathematics as a process that one "does" to numbers. High-press questioning is a means of shattering this misconception. Questioning drives at the meaning behind the procedures, shifting the focus from memory to understanding. Discourse is then the means of bringing this out and engaging the students in making meaning from the mathematics.

The second finding was curious in that it did not seem to correspond with the first. The initial survey revealed that 90% of the students thought it was OK to make mistakes in their work. This seems to fly in the face of student values placed on speedy memorization and strong procedural ability. It indicated a comfort with mistakes and a willingness to try, which might have allowed students to examine more challenging material. The survey also showed overwhelmingly that one of the reasons why students tried in math was simply to learn new things. They had a natural curiosity about mathematics. This was not their only reason for putting forth effort, but it was a significant factor. The students proved their openness and willingness to try new things by adapting to the groupwork style of the class and the expectations around discourse and justification.

The third finding from the survey concerned student motivation. Though students confirmed (97%) that they were encouraged to try new things, 84% often or always tried to learn new things in order to get a good grade. This response was confirmed by another question on the survey which indicated (50% strongly agreeing and 40% agreeing) that effort in math class was attributable to the desire to get a good grade. Most agreed that it was important to use the teacher's method, though more than one-third said it was not important. This showed itself in class as many students struggled to come up with their own entry into problems. Perhaps as a result of these two forms of external motivation (grades, teacher approval), many students were initially resistant to become authorities of mathematical understanding. They were accustomed to having someone else tell them they were either correct or incorrect. During most classes, I shifted the responsibility back to the students to determine mathematical correctness.

The second survey, given after the completion of the study unit and one additional unit taught in the same manner, showed some significant changes. Table 3.1 highlights the changes in student dispositions.

Table 3.1. Changes in Student Dispositions

Percent Increase	**Item Description**
30% +	• It is not important to appear smart to others • Success in math is mainly about thinking for yourself
20%-30%	• It is not important to use the teacher's method or get more answers right than classmates • Math is not just procedures to be memorized
10%-20%	• School math is based on things that happen in the real world (rather than disconnected) • In math it is ok to make mistakes

Taken together, these changes point to a reduction in competition inside the mathematics classroom. The nature of cooperative groupwork implemented in my classroom seems to have made a great impression on the way students viewed success in math.

The survey also asked student to rank various of aspects of mathematics. Five items were ranked with "1" being the most important and "5" being the least important. Data from both classes was compiled and averaged to determine the rankings in both pre and post surveys. Though the order of the rankings did not change significantly, there was a strong current of change moving through the data. Table 3.2 highlights the post survey results and the movement in the rankings.

Table 3.2. Student Rankings of Mathematical Aspects

Rank (previous)	**Average Rating**	**Change**	**Item Description**
1 (2)	2.03	-0.39	Understanding big ideas
2 (1)	2.29	+0.71	Memorizing facts and rules
3	2.39	-0.50	Helping others learn
4	3.91	-0.49	Finishing lots of work
5	4.37	-0.33	Learning to use calculators

The only item to lose importance was “Memorizing facts and rules.” This was also the largest overall movement in the rankings. We can see that memorization is still an important aspect of mathematics to the students, but it has fallen from its previous heights.

The initial survey revealed that these 5th grade students viewed math as a set of memorized procedures that were handed down and then assessed by the teacher. Students showed a willingness to make mistakes, but the desire to learn new things was often for the purpose of getting good grades. At week twelve, students placed less importance on memorization and more importance on cooperating to understand the big ideas in mathematics. The majority of students felt that math was based on things that happen in the real world, and that success in math is mainly about thinking for yourself. Most students still wanted to earn good grades, but there was a significant decrease in the importance of appearing smart to others. The survey results indicate that students felt it was important to have a strong conceptual understanding of mathematics. Table 3.3 provides a summary of findings from the student survey.

Table 3.3. Student Survey Findings

Week 1	**Week 12**
1. Mathematics was seen as highly procedural. Memorizing facts and rules was seen as the most important goal in mathematics. 2. Students thought that making mistakes was OK and they were encouraged to try new things. 3. Almost all students said they were motivated by the desire to get good grades and most students thought it was important to use the teacher's method.	1. Conceptual understanding overtook memorization as the most important aspect of mathematics 2. Cooperation and thinking for yourself gained in importance to the students 3. Students felt much less pressure to appear smart in front of their classmates

High-press questioning. Teacher questioning in whole-class settings served as a model for the kinds of answers that would be acceptable as justifications. A norm was developed in which students were not able to give an answer without conveying the conceptual reasoning for that answer. In this way, students learned how to question one another and what it meant to provide adequate proof for their answers. All students in the group benefited from hearing how others conceived of mathematics, and each student had to explain their thinking, or the thinking of the group, when pressed. As a result of teacher questioning and student discourse, these students had numerous opportunities on a daily basis to listen to and use the language of mathematics in the context of their classwork. These interactions afforded them opportunities to deepen their conceptual understanding of the material. I also learned a great deal about what each student knew or was struggling with as they attempted to explain their thinking.

The video recordings of classroom groupwork presented a different perspective on my teaching. Most videos only captured the work of one table group (just 3-4 students), but they captured it for an entire period. The stationary camera often blended into the background as there was no one who operated it. It was fascinating to see the classroom from a second perspective. One week I sat off to the side of the camera and recorded different students at the document camera as they explained their thinking after working on a problem with their table group. Another week I went around the room with a small handheld video camera and talked with a few different groups — revisiting each during the period.

During the first month of school I did a great deal of talking to the students about routines and expectations both in and out of math class. I waited to begin the study until the eighth day of school, but there was still a significant amount of training that needed to occur during this unit that did not occur in subsequent units. Almost 17% of the teacher codes I made in the transcripts of recorded classes were clarifying my expectations and establishing routines. Still, I found that just over 50% of the teacher codes tallied were of me questioning or pressing for student thinking. Questioning could turn into a press if it was sustained questioning, or if the question was asking a student to consider a deeper conception. The discovery of a generalization might come about through a series of questions that press student understanding.

In a video from the first week I asked: "How do we know if 4 goes into 30?" I wasn't expecting this to be much of a dialogue, but it turned into one. I had just talked about justification and the need for it to meet an agreed upon standard that we would work out as we went along, so students were trying to provide reasons for their thinking. One student thought 4 did not go evenly into 30. Instead of saying that she was correct, I asked her how she could be sure.

Amber: Because 4 is an even number.

Teacher: [I repeat her answer aloud.] That's true. Why does that make it not go into 30?

Amber: Because 3 is an odd number.

Teacher: So 4 won't go into a number with an odd first number? [Think time] Doesn't 4 go into 16? And 1 is odd right? So that won't hold up as a good justification, but let's keep thinking about this.

This was an interesting discovery of student thinking and one that I would have dug into more in the moment had it been a one-on-one situation. Each year I have been surprised to find the concepts of even and odd numbers are not firmly in place for all of my fifth-grade students. We had spent some time discussing even and odd numbers about ten minutes before this when we wondered whether or not 2 was a factor of 30. I restated the question and asked the class again.

Kelsey: Well, it's like 4 doesn't go into 30 because it doesn't go into 15. And 15 times 2 is 30.

Teacher: Huh, let me think about that. [I repeat her answer aloud, still thinking.] Can anyone think of an example that would make that argument not work? [One second of think time has elapsed and I begin talking again.] Does 4 go into… [Think time… and then I restate her conjecture. There is think time for the students theoretically, but they seem to be watching me do the thinking.] So 4 has to go into half of the number?

Kelsey: Well...

Teacher: Like 4 goes into 12 right?

Kelsey: Yes.

Teacher: Does 4 go into 6?

Kelsey: No… not evenly.

Teacher: No, not evenly. But it does go into 12?

Kelsey: Yeah.

Teacher: So because 4 doesn't go into 15, we can't necessarily say that it doesn't go into 30.

This student has made another interesting observation, but did not have all of the pieces put together yet. She could have proved it by saying that 4 doesn't go into 30 because 2 doesn't go into half of the number, but I didn't think that this was her line of thinking. This kind of proof was only later understood at the end of the unit. Another student then raised her hand to supply a third attempt at justification.

Cassidy: It does go into 32, which means that it couldn't go into 30.

Teacher: So you're saying that it does go into 32, which means it couldn't go into 30 - why?

Cassidy: Because it also goes into 28.

Teacher: So that's just 2 away, so there's not enough space for it to go in right? Now how do you know that it goes into 32?

Cassidy: Because 8 times 4 is 32.

Teacher: 8 times 4 is 32 - that's just a known fact. This is valid. This is a valid proof. You could say that "I know that 8 goes into 32 so it's too close to go into 30." Or you could say "I know that 8 goes into 28, [and] it's only 2 away."

Within the exchange there are examples of both high- and low-press questioning. Without the high press, the students would not have an opportunity to express their own conceptions, and without the low press, my line of thinking would have been obscured. Table 3.4 provides a comparison of these two types of questions.

Table 3.4. High- and Low-Press Questions

Questioning Level	**Examples**
Low Press (Funneling, Explicit, Closed)	• Doesn't 4 go into 16? • And 1 is odd, right? • Does 4 go into 6?
High Press (Focusing, Implicit, Open)	• How do we know if 4 goes into 30? How can you be sure? • So 4 has to go into half of the number? • It does go into 32, which means it couldn't go into 30 – why?

The class worked through one example of what it means to provide valid reasoning scaffolded by my teacher questioning. The first outcome of this questioning was that the students got to explain the mathematics. It took time to develop these norms, and I had to constantly remind myself to get out of the way, but even this innocent example allowed me to model mathematical thinking, justification, and questioning. Through a series of student attempts, the class was able to formulate what it meant to answer in a way that showed conceptual understanding. There were certainly many stones to turn over in the ground we covered, but I looked to do that in table group settings. I certainly didn't give enough think time for the students on some of the questions, but by modeling what I perceived to be "student thinking," I hoped to establish my expectations for questioning and justification between students. This is a good example of Level 2 discourse as defined by Huffered-Ackles et al. (2004) (reviewed in Chapter 1).

A corollary benefit to student talk was that I spent a great deal of time listening to student conceptions. This served as a minute by minute, or person by person formative assessment. During the course of conversations with groups it was easy to identify students who did not have a strong grasp of the group's work. In these moments I checked in with that student and either pressed for more information, or provided more time by leaving the group with the promise that I would be back to hear how he or she could explain the work of the group. On the other end of the spectrum, those students who had a strong grasp of the material could be challenged on the spot to go further with the material. The outcome of listening to students was that I knew where they were struggling and where they were strong. Coincidentally, the fourth week of the study also happened to be conference week. They were intended as planning conferences as we had only been in school for twenty-two days, but as a result of listening to each of the students in class I had a very good sense of how they were all doing. These conferences had always been slightly uncomfortable in past years as I felt rather uninformed in my first interaction with parents. This year's conferences were markedly different. Students who in past years might have slid under the radar for much of the first term had already revealed to me in their own words each day in class just how they thought about the mathematics we were exploring.

I began the unit by modeling appropriate questioning and justification. By the end of this first week, I had established some of the classroom norms around student discourse. It was difficult at first to give appropriate think time and allow students to come to a group understanding of key concepts. Exposing student thinking required that I provide time for the conversation to take place without running it myself. This was how I perceived my students could best integrate new mathematics into their existing schemas. We needed to hear, discuss, and question any misconceptions along the road to new learning and there was no way for one teacher to facilitate all of those conversations in a whole-class conversation. It was by means of the table group conversations that I hoped this process could take place.

Student discourse. Students were more engaged with the material when they had to talk about the problems in small groups. I found that by having each student listen to and participate in small group discussions they were able to form well conceived justifications and explanations of their work.

When I listened to the students in class and watched them in the videos it was obvious they were talking a lot. I went into the analysis without a preconceived set of codes, and when I had finished revising and condensing my codes I had labeled 250 distinct student actions with 27 different codes. Because there were so many codes it was difficult to see a pattern in the data. I decided to group the codes that indicated some form of "student talking" around a mathematics task and found that 60% of the total codes labeled students engaged in mathematics discourse of one kind or another. Table 3.5 gives examples of each of these types of communication.

Table 3.5. Examples of Student Communication Codes

Code	**Types of Student Talk**
Student Talk	Students asking each other how they solved problems, discussing or reviewing current problems or strategies, checking group members' understanding. "So let's do the rainbow thing[6]. I'm going to do the rainbow thing - it's easiest for me."
Explaining Thinking	Students explaining how they know something, discussing what the numbers represent, why they made the choices they made "[4] does go into 32, which means it couldn't go into 30."
Restating Thinking	Agreeing with and restating, or adding on to what another person has said. After one student has answered a question about the overlap in a Venn diagram correctly saying that "it is the factors of both numbers," I ask another to restate this and she says that "they are common factors."
Justification	Proving something by logical argumentation "The square number [lockers] are open because the square numbers have an odd number of factors and non square numbers have even."

The largest code contributors were student talk, explaining thinking, restating another's thinking, and justification.

[6] The "rainbow thing" is a method for checking factor lists. If you list the factors in order and draw an arc connecting the factor pairs, the concentric arcs look like a rainbow.

The different types of student talk all contributed to the group's conceptual understanding. Whether they were discussing strategies, thinking through problems, or explaining their reasoning, students had opportunities to both speak and listen. Jamie and I began by just going from group to group somewhat randomly, but towards the end of the study we divided the tables (four apiece) and would sometimes switch groups partway through the period. In this way we could systematically and thoroughly monitor and take part in at least eight different conversations on the mathematics.

In the last week, I captured the kind of student discourse I was working on developing in the first week when we discussed why certain numbers were factors of other numbers. The table group I filmed was working on The Locker Problem (Appendix E), looking for patterns in the data they collected. In the problem all of the locker doors start out closed, but as the students run through the hall they open or close locker doors depending on their student number. Student 1 opens every locker because 1 is a factor of every number. Student 2 closes every second locker. Student 3 encounters some that are open and some that are closed, so this student just changes the state of every third locker. And so it goes for 1,000 students through 1,000 lockers. The focus group created a model of the first 30 lockers, adjusting the state of the lockers as the students went through. The following discussion concerns the third student to go down the hall of lockers.

Nick: So the student opens the door if it's closed and closes the door if it is open, so he changes the state of the doors 3, 6, 9... So he'll open all of the even numbers that are divisible by 3.

Matt: Are we ready for 3?

Naomi: No, let's just open all of the ones that end in 3, 6, 9, and 2. [She says the last digit of first few multiples of 3.]

Lydia: So what are we doing now?

Matt: So now we're on the one that opens every third.

Nick: So every odd number that is divisible by 3 you… no… no

Matt: Every number that is divisible by 3.

Lydia: Every third number… every third number…

Nick: Every odd number that is divisible by 3 we close it and every even number we open it.

Matt: So pay attention to what the number is.

Naomi: Yeah like 6 you have to open because 6 is already closed.

Lydia: We have to do 3 open… 3 closed. [This seems to refer to the fact that the third student will count three lockers and open a door, then count three lockers and close a door.] That's what we have to do.

The communication in this group is notable for the quality of the discussion and the balance of voices in the conversation. Sometimes they are working at different levels, but they all develop an understanding of the mathematics involved in the problem. This group discovered a pattern and stated it as a generalization: Student 3 will open all of the lockers divisible by 3 and 2, and close all of the lockers divisible by 3 but not by 2. It was said right away by Nick, but the others weren't ready to hear it yet. After a two minute conversation where everyone was involved, they all understood it. Nick then said that Student 3 will open all of the multiples of 6. A few minutes later, after continuous discourse on this problem, another generalization was made.

> Nick: So if it's at 4 and it's open… or no… no they're all going to be opened. [1 opened all of them and 2 closed all multiples of 2. Nick is trying to see how 3 impacts the doors for 4, but isn't quite there.] We're just doing… they're all going to get opened. Wait no. So we're doing multiples of 4. So, if there is a multiple of 4 that is also divisible by 3 then we close it. But if it is just a multiple of 4 then we open it.

The group was quiet as they considered this conjecture, but after a few moments they began to examine their models and see this interesting relationship. There were three adults in the room at the time walking between the table groups, listening in and asking group members questions, but no one was at this table at the time. Though the class could have listened as the teacher led this type of conversation, when it happens spontaneously, coming forth from the students as they make their own connections in the mathematics, it seems that there is a deeper level of learning taking place. The students owned these ideas. This is the type of discourse that takes place in a Level 3 classroom (Huffered-Ackles et al., 2004). These students, as Van de Walle (2010) would say, are *doing mathematics*. Students were not explicitly asked to make these types of generalizations, and most groups probably did not, but the freedom to take the conversation in a direction that was meaningful to the group allowed for a very rich classroom experience. During the final five or ten minutes of class I often summarized the learning that took place. As I summarized the different approaches that occurred around the room, the students gained an insight into the reality that there were many different ways that people think about mathematics, and my hope was that this helped dispel the myth that mathematics is mainly about memorization and procedures. Mathematics is more about understanding principles than following rules. There were many of these types of interactions in the table groups. I found that the students were much more active when they were engaged in mathematics as a group than they were when I was leading a discussion with the entire class. I have taught this unit about eight times prior to this year, and I felt that this group of students, having had a much greater access to discourse and having been pressed to justify their thinking, had a much deeper conceptual grasp of the material.

Assessing conceptual understanding. After my initial coding, I reviewed the video transcripts for students expressing conceptual understanding. It would be misleading to look too closely at trends or differences between the two classes, as the videos typically only captured one group, and it was not always the same group from week to week. There was however a very clear difference in the classroom videos from the beginning of the study to the end of the study. Because of a movement towards less teacher talk, less time establishing routines, and richer tasks that the students built up to over time, there was significantly more discussion and conceptual understanding evident in the videos towards the end of the study. In fact, the average number of codes for conceptual understanding in the fifth week was almost double the average for the first four weeks.

I intended to measure comprehension with weekly assessments, but the more I thought about the nature of the assessments themselves, the less the scores seemed to mean. The cognitive demand of each quiz was variable, yet the relative weight of each was nearly identical. Individually the quizzes didn't paint a very complete picture, and the trend created by looking at the group of assessments over time might have said more about the assessment than the student. Still, the assessments should be able to say something about student comprehension.

My teaching approach was identical for both classes, so I was curious to see how the two randomly created classes would compare. The average scores on the weekly assessments tracked almost identically (Appendix F), and the overall percentage grades for the unit, which included homework, differed by just one percent. When I averaged all of the quizzes, the trends disappeared - leaving only a mean score. This mean score was then a function of the various types of questions taken across the entire unit with varying levels of cognitive demand. The two classes had an average score for the quizzes that was within two-tenths of a percent of each other. Rounded to the nearest half of a percent, both would be 77.5%. The average scores on the final quiz differed by 7.26%, which was the largest difference in any of the assessments. As it turned out, the class that did better was given an extra 20-25 minutes to work on this assessment. The combined average on the final assessment was the highest of any week at a little over 90%. In general, what can be said about the results of these assessments is that the students showed a fairly strong grasp of the material, and that the classes performed equally well on them.

On the back of the quizzes for week 3 and week 5 I asked for a self-assessment of how the students were feeling about the unit to that point and about the quiz itself. On the first one I had students write responses by hand; most student used key words from a verbal list of adjectives I provided. This became a rubric by which I scored their responses and which I then gave to the students for the second self-assessment. The final rubric is shown in Figure 2. This data taken together with the information gathered from the weekly quiz scores helped me better understand student comprehension.

Figure 2. Self-reflection Rubric Given in Week 5

What is your comfort level with the material we have been working on in class?
How did you feel taking this quiz?

(This does not affect your grade)

0 – I have very little confidence about this stuff
1 – I'm shaky, worried, or struggling a bit
2 – Confused about **some** things
3 – Pretty comfortable with most of the concepts
4 – Very comfortable, more please!

One class averaged 2.85 in week 3 of the study and then dropped to 2.69 on the final quiz. The other class averaged 2.77 in week 3 and then went up to 2.92 at the end of week 5. This second group was the class that was given extra time on the final quiz. The average self-assessment score indicates that the students felt fairly comfortable with the material. This measure triangulates and confirms my finding that the students demonstrated conceptual understanding in their classroom discourse and on the weekly assessments.

I felt that the students had a good grasp of the material, and I would say that the students generally assessed themselves as fairly comfortable with most of the concepts. The assessments themselves showed students had developed conceptual understanding. Still, the central component to my assessment of the student's understanding of the material was their ability to discuss the problems in class and answer questions verbally. When students claimed that it was "difficult to explain," I knew that there was still work to be done. The high-press questioning and student discourse prepared them to be successful in this light.

Summary of Findings

High-press questioning quickly became second nature as a means of communicating with my students. The students accepted this way of working in the math class and developed the habit of explaining their thinking when asked about a problem. During groupwork I strove to scaffold the quality of student discourse through their use of questioning and justification with one another. As I analyzed the data and reflected upon my teaching I was able to draw a number of conclusions. I came up with findings based on various data sources that either assessed current thinking or tracked the development of student understanding. I also noted some outcomes of the pedagogy that I implemented.

Generally, I found that by approaching students with meaningful questions I was able to illicit meaningful answers. I found that as classroom norms for discourse and justification established themselves, students were able to work in groups productively for the bulk of the period. This allowed for many more conversations and many more teachable moments than I could have facilitated from the front of the room. My students felt fairly comfortable with material that has in the past presented numerous cohorts with challenges. It is my contention that the process of incorporating as much on-task student talk into the classroom as possible allowed for a deeper conception of the material. A summary of my findings is as follows:

- Student dispositions became more flexible, progressive, and cooperative. Students seemed less caught up in the mechanics of mathematics and came to view the discipline more holistically. Conceptual understanding of the big ideas emerged as the most important aspect of mathematics.
- High-press questioning led to student discourse which then created opportunities to develop conceptual understanding. Classroom norms requiring justification for student responses brought conceptual understanding to the forefront. I continuously monitored small group conversations and interjected high-press questions to push student thinking forward. Students were expected to be able to articulate how and why they thought the way they did.
- Student discourse provided individuals with many opportunities to express their own thinking and listen to the thinking of others. This helped them hear and practice vocabulary, but more deeply it gave them opportunities to put their own language around mathematical conceptions.
- Student discourse also allowed me to find out more about my students' abilities in math more quickly than ever before. Listening to student conceptions throughout the period became an effective formative assessment.

> Knowing how the students thought about the mathematics allowed me to better guide the pace of classroom activities and address student needs.

These findings support the efficacy of teacher questioning and student discourse in the effort to improve conceptual understanding of mathematics in students. My implementation was by no means perfect, but there are a number of implications for my classroom teaching practice.

Chapter 4: Conclusions

Connections between my research and the literature

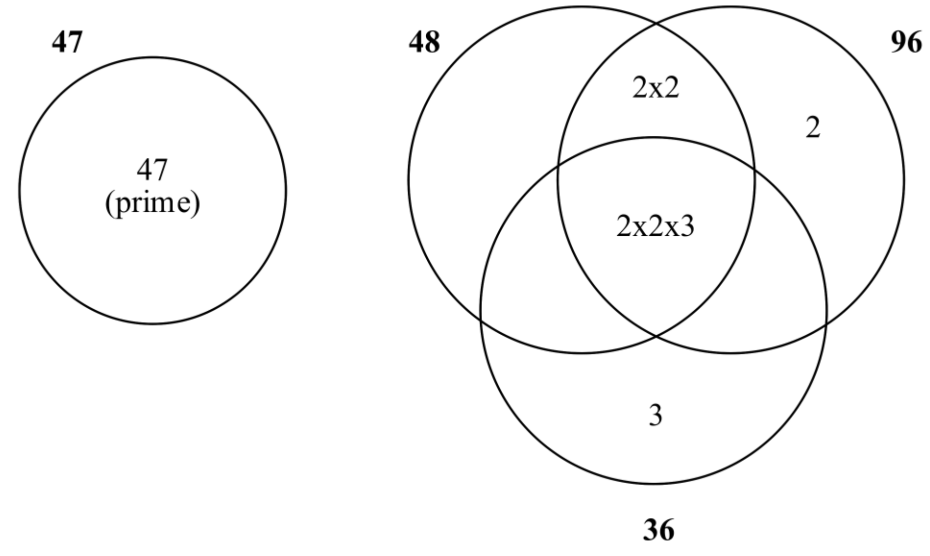
My research examined two distinct aspects of a mathematics classroom that revolved around the student-centered pedagogies of student discourse and high-press questioning. First, I wanted to find out how high-press questioning affected students' conceptual understanding of mathematics. Second, I wanted to learn about the teacher's experience implementing this student-centered pedagogy. I found numerous studies on the impact of questioning strategies with students. The findings from my own study confirm the efficacy of high-press questioning as a means of helping students develop a strong conceptual framework in mathematics. As for the teacher's experience implementing this pedagogy, I found very little research done by teacher-researchers. My experience teaching through questioning and student discourse was very positive. I believe that it helped facilitate many of the best practices in teaching that previously seemed disconnected and difficult for me to realize.

Student discourse and learning. Research into student learning has shown that teaching students how to do math through the memorization of procedures yields very little conceptual understanding (Boaler & Greeno, 2000; Phillip, 2000). Despite this research, most classrooms utilize direct instruction as a means of teaching low level skills rather than big ideas (Ball, 2001). My research was directly focused on the outcomes of a student-centered pedagogy that used group discourse as a means of developing conceptual understanding. Having students in my study grapple with big ideas in mathematics and providing opportunities for student discourse made the material accessible, relevant, and coherent. The learning cycle begins with students accessing prior knowledge (Bransford et al., 2000). When students are asked to discuss a problem in groups, they naturally engage their prior conceptions, listen to those of other group members, and revise their existing schemas where necessary. Because students were put into the position of having to regularly problem solve and communicate their thinking, they were encouraged to take ownership of their mathematical reasoning. This personal connection to mathematics established their membership in the mathematical community. The NCTM paradigm for mathematics instruction centers around reasoning and making sense of mathematics. Getting students to explain their thinking opens the doorway to this type of sense making.

Like the students who learned specialized vocabulary and appropriate justification in Khisty and Chval (2002), my students developed similar habits of mind. The regular practice of student discourse in my classroom allowed our math community to expose misconceptions and replace them with more accurate models of thinking (Donnovan & Bransford, 2005). Expecting students to learn mathematics by sitting through a lecture is probably about as effective as expecting them to learn how to play basketball by watching a game. There are certainly things to be gained by watching, but learning is best done through participation. Taking part in the activity provides a richer learning environment, and it's much more fun for the students. Kilpatrick, Swafford, and Findell (2001) describe a student's productive disposition as a "habitual inclination to see mathematics as sensible, useful, and worthwhile coupled with a belief in diligence and one's own self efficacy" (Kilpatrick et al., 2001, p. 5). Students in my classroom generally displayed productive dispositions towards mathematical tasks when given the responsibility to carry out meaningful work as a team. This then yielded a shift away from seeing math as simply memorization and procedure towards an emphasis on conceptual understanding and cooperation. Further, the students showed proficiency in classroom-based group tasks and individual assessments requiring conceptual understanding and procedural fluency during the study period and beyond. These findings support work done by Herbel-Eisenmann and Brefogle (2005) and Wood (1998) on the development of cognitive abilities through focusing questions. They also confirm those of Kazemi and Stipek (2001) in which the high-press questioning techniques promoted conceptual understanding.

I agree with Yackel and Cobb (1996) that a foundational component for the successful implementation of these pedagogies can be attributed to the creation of sociomathematical norms, which yielded a community of practice around the serious exploration of mathematics in my classroom. The behavioral norms within my classroom centered on the communication of students' understanding, expressed both verbally and in writing, using the formal language of mathematics. Just as Kazemi and Stipek (2001) found, questioning opened up opportunities to reconceptualize students' thinking. Collaboration allowed my students to discuss their reasoning and it provided individual accountability for knowing the material (Kazemi & Stipek, 2001). Listening to others was essential as group work was the standard method of working in my class, and I could be counted on to inquire into each student's thinking. Students were expected to make sense of differences in the way various group members attempted to solve problems and the final results they achieved. My study confirmed what Bransford et al. (2000) discussed and Wood (1999) found: this form of dialog had the potential to put students in a state of disequilibrium which when resolved, led to new learning. In a very real way, this practice helped reshape my students' existing knowledge as they were forced to reconcile mathematics into a body of knowledge that needed to be explained, connected, and understood. For example, when we explored prime factors, students were given a new insight into the nature of numbers. 48 was no longer a static number closely situated to 47. When viewed as the product of primes, 48 looked nothing like its neighbor; it suddenly appeared to have much more in common with 36 and 96 (Figure 3).

Figure 3. Prime factor relationships



This new knowledge reshaped some of their existing knowledge. Crucially, members of the community of learners were participating in a reimaging of their mathematical identities (Lave, 1996). This is the process of one's trajectory affecting one's present state. To repeat Lave, "who you are becoming shapes crucially and fundamentally what you 'know'" (Lave, 1996, p. 157). This profound philosophical statement reminds me of the transformative potential of teaching and learning: when students see themselves participating in meaningful math discussions and hear themselves and others using the formal language of mathematics, they have become true practitioners of this specialized discipline (Huffered-Ackles et al., 2004; Khisty & Chval, 2002; Lave, 1996; Mewborn & Huberty, 1999; Walshaw & Anthony, 2008).

Research on high-level student work done by Henningsen and Stein (1997) focused on the factors that most powerfully influenced the engagement of students. The most influential aspects included: building on prior knowledge, high-level performance modeling, scaffolding, sustained pressure for explanation, and providing appropriate time for tasks. In my study, each of these aspects seemed to flow naturally from a pedagogy that focused on student discourse and meaningful justification. As my students learned how to discuss math through my questioning and modeling of appropriate justifications, they activated prior knowledge. The questioning served to both press for explanation and scaffold their understanding. During tasks, I monitored the progress of the class and modified the time for a given task (by group) as needed. In this way, groups whose members came to a sufficient understanding of the material were pushed forward while others who needed more time were provided with it. I was freed from the single lesson, single pace dilemma that does not meet the needs of all students.

Teaching from the sidelines. Teaching and learning are symbiotic rather than distinct processes. It seems difficult to claim that I have taught something if it has not been learned. In my research, I wanted to document the teaching experience of using questioning and student discourse as the central means of learning. Removing myself from the center of attention was slightly unsettling, and the release of control to the students over how they would come to know the mathematics seemed almost irresponsible at first glance. But what I found was that my experience "on the sidelines" was much more powerful than it would have otherwise been "on the stage." I became the coach, rather than the quarterback. I was free to see the entire classroom, to step back and enter group conversations where I perceived I most needed to be. I was able to attend to each group and discover how students thought about and verbalized their understandings. This would not have been possible with direct instruction. Referring back to Figure 1 (Chapter 1), we can see that the arrows, representing communication or interaction, come from many directions rather than simply from the text and teacher to the learner. My implementation was by no means perfect and there were many times I was not able to be everywhere I wanted to be at once, but it was a far greater learning environment for my students.

Recommendations and Conclusions

I found that student discourse and high-press questioning provided more opportunities for more students to develop a meaningful understanding of mathematical ideas. I will continue to use these teaching practices in the future. These practices required that I restrict myself from answering high-level questions, deferring to students to articulate and refine their ideas — sometimes through asking them questions of my own. As I became more practiced with asking questions, this restriction became second nature. If given a chance, students were able to explain concepts we were working on, and if not, I was able to find questions to help them organize their thinking. Nothing about the implementation of these practices stood out to me as something I needed to change, but I'm sure there will always be new ideas which I will want to incorporate. With each year, and in each grade, there are new variables that demand a teacher's attention.

One of the essential elements of the student-centered classroom is the arrangement of desks. Table groups greatly enhance face-to-face conversation, argumentation, and the completion of complex group tasks (Boaler, 2002; Weber et al., 2010; Wood, 1999). I know I would struggle to teach in a classroom without them, where seats are bolted to the floor and the free exchange of ideas is hampered by poor classroom design. Student seating assignments are also an important consideration for the teacher. I used randomly generated seating assignments which allowed students to change groups regularly, quickly, and without imposing myself into the process. Boaler (2002) highlights the importance of mixed ability classrooms and groups. The make-up of my two math classes and the near weekly table group changes was random by design. Sometimes groups were generated that I would not have put together myself, and sometimes the most unlikely groups surprised me with their ability to work together.

One of the most significant findings from the student surveys was that students discovered the importance of thinking for themselves. Students let go of the fear of "looking stupid" in front of others as the value of student talk became established. I agree with Walsh and Sattes (2004) that good questions help students think. When students reflected on a question and tried to explain their thinking, they found that the question itself was a way into understanding something about the concept. But the most impactful change to student dispositions was not conceptual understanding as I would have expected. Rather, it was the elevated value students placed on cooperation. This tells me the discourse that took place was meaningful.

I don't think the groupwork would have been as successful without the establishment of sociomathematical norms. This is what framed the student discourse, and allowed students to switch groups but maintain an understanding of how to work together (Yackel & Cobb, 1996). One of the areas I would be interested in exploring in the future is how complex instruction (Cohen, 1994) can play a role in the norms and the resulting group dynamic. Would more structured table-group norms, especially the use of roles[7], create a more equitable learning environment? A related area that I would like to explore more deeply is that of status within the class and/or group. Status emerged on a regular basis, and I sometimes found ways to assign competence or diminish dominance, but it was

[7] Cohen (1994) describes developing and using roles in groupwork to delineate responsibilities between group members. This is intended to improve student engagement and foster equity.

not a central focus of the pedagogy. Justification will continue to play a central role in the discourse, in that our understanding of key math concepts and our ability to explain how they relate to one another is paramount (Cobb et al., 1992; Kazemi & Stipek, 2001; Khisty & Chval, 2002; Yackel & Cobb, 1996).

Class size is an important consideration for the implementation of this type of instruction. During the study period I taught two classes, each with over thirty students. I had students work in groups of 3-4 which generally meant eight groups each period. Sometimes it was a challenge to monitor the discussions and provide teacher questioning, both high- and low-press, to facilitate student learning to so many groups. Having a paraprofessional in the room made it so that the number was well within our grasp as we divided the room between us. Had it not been for my peer, I would have felt less confident that I could maintain high-level student work in eight separate table groups of students. In a lecture hall it does not matter if there are 10 or 100 students listening, but class size matters when teachers need to actively involve the students. It became quite clear to me that the debate over class size has a very real impact on my ability to provide the optimal learning environment. In my estimation, 24 students in a fifth-grade classroom is about the limit for a single teacher to meaningfully engage each student in a single period when using small groups. Small class sizes, even beyond the primary grades, are very closely related to the quality of education teachers can provide for their students. Nonetheless, I would still attempt groupwork and use student discourse with large classes despite my waning ability to be involved in each discussion. Student discourse is such a powerful formative assessment that even without listening to all of the conversations, listening to just a few would still inform me as to the extent of the learning taking place.

Doing research benefited my practice both immediately and in the longer term. The regular analysis of student thinking greatly enhanced my first implementation of this pedagogy. Teachers are always collecting data, but not to the degree that a researcher does. Conducting my own research also helped me approach my students as a problem solver, a learner, and an experimenter. Franke and Kazemi (2001) found that students learned more mathematics being taught by teachers who focused on students' mathematical thinking. Further, they found that approaching student learning in this way helped teachers continue their own learning (Franke & Kazemi, 2001). Systematically examining student thinking through questioning and discourse is a practice I intend to continue. After collecting and analyzing paper copies of a student survey, I realized that I could create the same survey online and have the information populate a spreadsheet. I created the form in less time than it took to format the Word document and plan to give it to my students at least twice a year in the future. Given the speed at which I will be able to collect and then analyze the spreadsheet data, using this research tool will help me continue my investigation into these practices.

Unanswered questions

It is frustrating to reduce all of the hard work that goes into a year of teaching and learning into a single test, but achievement seems like the emphasis these days and I don't see standardized testing going away any time soon. The focus of my research was conceptual understanding, and though the tests attempt to measure this, I am not quite sure that they do. There are things to be seen in the scores, but they tend to take the focus away from the real purpose of education, substituting performance for understanding. Additionally, some districts have moved towards a merit based pay system for teachers, which would undoubtedly reinforce the importance of the tests by rewarding teachers for high test scores. With so much attention being paid to year-end standardized tests results, I would be interested to know the impact of this pedagogy on student scores. I also wonder whether or not those results would be reliable from year to year. I do not let these tests dictate the way I teach my students, but I am forced to live with the reality of their weight. I want the best for my students, and that includes being able to pass the state's measure of the content that I teach.

In my classroom, students knew how to work in groups. When they were asked a question or given a task, they generally understood what was expected in the form of a response. I wonder if the sociomathematical norms we followed were internalized for the students, or whether they were dependent on my presence. Is there transfer to other teachers and other subjects? I would be interested to know how students value conceptual understanding in other areas of school. Additionally, I am curious about the long term implications of this type of pedagogy. It may be practiced by a few teachers during a student's K-12 experience, but the majority of classrooms will probably feature direct instruction. Would there be any lasting advantages for students who have learned to function well in a student-centered classroom?

Areas for future action research

By using student discourse as the means of developing conceptual understanding in mathematics, I may have created challenges for students who had difficulty in social situations. It is likely that the instruction was less effective for them than it was for others. I would be interested to know what types of students struggle with group work and student discourse, and what strategies prove effective for helping them be successful. I believe that students need to be able to listen to others and explain their own thinking, so to me it is not a question of *if* this pedagogy can be used by students who struggle socially, but *how*.

Finally, I would be very interested to see research focused on procedural fluency using the methods I practiced to target conceptual understanding. Kilpatrick (2001) defined procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately" (Kilpatrick et al., 2001, p. 5). Procedural fluency is not simply a matter of carrying the one, borrowing, or lining up the decimal point. Procedural fluency under this way of thinking reveals a very sophisticated understanding of the various methods for solving a problem and the skills to both choose the efficient path and then to carry it out accurately. During the course of the unit immediately following the research unit, the students were studying fractions and learning how to compare them. I taught the students five distinct strategies for comparing fractions and created a code for each one, which the students would text[8] me after they wrote their answer. In

[8] The codes reminded me of text messaging shorthand, which often uses acronyms. Students did not actually text me, but rather wrote in the "text" as a means of justifying how they thought about the fraction comparison. The strategies and their corresponding codes were: Same Denominator (SD), Same Numerator (SN), Benchmark (B), Piece Size (PS), and Common Denominator (CD).

class, students were asked to justify why they chose a particular strategy. This was a small insight into this idea.

Closing comments

When I was considering my research topic I thought back to reading about the brain and the learning cycle. I thought about language acquisition and how integral language is to thinking. It occurred to me that if I could change anything in my classroom it would be that the voices of the students would be heard from the hallway rather than my own. Not a single proud voice with an answer ready, but a chorus of questioning voices that struggled together to make meaning for themselves and for one another. Teaching students to value cooperation was not my intention, but it does seem like a natural outcome of the work with which we engaged. The level of conceptual understanding reached by these classes through discourse and teacher questioning seemed on the whole far superior to any previous attempt I have made with this unit. We need to have students talking in our classrooms to facilitate their understanding of the formal language of mathematics, to provide opportunities for that language to hold the concepts we are asking them to learn, and most importantly for the learners to take up membership in the larger world of mathematics.

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Appendix A

Questioning from Plato's Meno
The following is an interchange between Socrates and Meno's slave who is referred to as boy.

Plato (380 B.C.E.) Meno (Benjamin Jowett, Trans.). *The Internet Classics Archive*. Retrieved from http://classics.mit.edu/Plato/meno.html

Soc. Tell me, boy, do you know that a figure like this is a square?

Boy. I do.

Soc. And you know that a square figure has these four lines equal?

Boy. Certainly.

Soc. And these lines which I have drawn through the middle of the square are also equal?

Boy. Yes.

Soc. A square may be of any size?

Boy. Certainly.

Soc. And if one side of the figure be of two feet, and the other side be of two feet, how much will the whole be? Let me explain: if in one direction the space was of two feet, and in other direction of one foot, the whole would be of two feet taken once?

Boy. Yes.

Soc. But since this side is also of two feet, there are twice two feet?

Boy. There are.

Soc. Then the square is of twice two feet?

Boy. Yes.

Soc. And how many are twice two feet? count and tell me.

Boy. Four, Socrates.

Soc. And might there not be another square twice as large as this, and having like this the lines equal?

Boy. Yes.

Soc. And of how many feet will that be?

Boy. Of eight feet.

Soc. And now try and tell me the length of the line which forms the side of that double square: this is two feet-what will that be?

Boy. Clearly, Socrates, it will be double.

Soc. Do you observe, Meno, that I am not teaching the boy anything, but only asking him questions; and now he fancies that he knows how long a line is necessary in order to produce a figure of eight square feet; does he not?

Men. Yes.

Appendix B

Student Math Survey (Boaler, 2008)
Your answers will be kept strictly confidential

1. How much do you agree with these statements about math: (check one box)

	strongly agree	agree	disagree	strongly disagree
Math will be really important in my future career				
Other subjects are more interesting than math				
Math is really useful in life outside of school				
Math is a lot of procedures that have to be memorized				

2. Which of these statements do you agree with MORE: (check one box)

School math is based in things that happen in the world ☐
OR
School math is very different from things that happen in real life ☐

3. Which of these statements do you agree with MORE: (check one box)

Success in math is mainly about memorization ☐
OR
Success in math is mainly about thinking for yourself ☐

4. In math class, how often do you:

	never	seldom	sometimes	often	always
Try to help your classmates solve a problem?					
Try to learn things because you want to get a good grade?					
Try to learn something new even when you don't have to?					
Try to get more answers right than your classmates?					

5. How much do you agree with these statements about math: (check one box)

	strongly agree	agree	disagree	strongly disagree
It is important to use the teacher's method				
It's OK to make mistakes in work				
It is important to avoid looking stupid in front of others				
Students are encouraged to try new things				
It is good to make mistakes at the board				

Over

6. I really enjoy math class when:

	strongly agree	agree	disagree	strongly disagree
The problems make me think really hard				
I am the only one who can answer a question				
I don't have to work hard				
The whole class learns together				
I am the first one to get a question right				

7. When I try hard in math it is because:

	strongly agree	agree	disagree	strongly disagree
I want to get a good grade				
The work is interesting				
I want to learn new things				
I want my classmates to think I'm smart				

8. Put these aspects of math in order of importance -
put a 1 by the most important, a 2 by the 2nd most important etc.

	Memorizing facts and rules
	Learning to use calculators
	Understanding big ideas
	Finishing lots of work
	Helping others learn

9. Describe an idea you thought was really <u>interesting</u> in math class:

10. Describe a <u>really good math lesson</u> you have had (in this class or any other) - saying why it was good:

11. Describe a <u>really bad math lesson</u> you have had (in this class or any other) - saying why it was bad:

12. What helps you learn in math?

Appendix C

Student Interview Questions

Set A

1. How would you describe a really great math lesson?

2. What makes math interesting to you?

3. What are the qualities that make someone successful in math? Who do you think is good at math? Why do you think they are good?

4. Can everyone be successful in math? What experiences have made you think this?

5. How do you feel when you get stuck in math? What types of things do you do when you get stuck?

6. Is there anything else I have forgotten to ask you that you would like to tell me?

Set B

1. How has 5th grade math been so far this year? What is different from previous years? What is the same?

2. How do you like working in groups? What makes a good group? What do group members do? What does the teacher do? Do you prefer groups or working individually? Why?

3. How do you feel about explaining your thinking to other people? Do you learn when other students are explaining their thinking? What works well? What does not work so well?

4. Does math class have different "rules" about how you are supposed to be in class? What makes math different?

5. How do you feel about asking questions in math class? How do you feel when you don't understand something and nobody else is raising their hand? If you have a question on your homework what do you do?

6. Is there anything else I have forgotten to ask you that you would like to tell me?

Appendix D

Fish/Young Math Questionnaire for Parents[9]

We are conducting a brief survey to find out what experiences people have had with math during their schooling.

There are certainly no wrong or right answers to this survey. Please answer as honestly and thoroughly as you can. We hope to use this information to help your child's experiences with math be even more meaningful than ours were.

If you should need more space, feel free to use another piece of paper.

1. What is math?

2. Do you like math? Why or why not?

3. What experiences did you have in math during your school years?

4. How is math important in our lives?

5. Is math important in your occupation? Why or why not?

6. How do you use math on a daily basis?

7. What is your favorite part of math? Why?

8. What is your least favorite part of math? Why?

9. What do you, or did you, find to be the easiest part of math?

10. What do you, or did you, find to be the most difficult part of math?

11. If you were having a problem in school with math, what would you have done?

12. When you got out of school, did you still see math as being important in your life?

13. Do you feel that math education is the same today as it was when we were in school?

Why or why not?

14. What changes, if any, would you like to see in math education today?

15. What words or pictures come to your mind when you think of math?

[9] Hubbard & Power, 2003, p. 69.

Appendix E

The Locker Problem (Lappan et al., 2009)

There are 1,000 lockers in a long hall of Westfalls High. In preparation for the beginning of school, the janitor cleans the lockers and paints fresh numbers on the locker doors. The lockers are numbered from 1 to 1,000. When the 1,000 Westfalls High students return from summer vacation, they decide to celebrate the beginning of the school year by working off some energy.

The first student, Student 1, runs down the row of lockers and opens every door. Student 2 closes the doors of Lockers 2, 4, 6, 8, and so on to the end of the line. Student 3 changes the state of the doors of Lockers 3, 6, 9, 12, and so on to the end of the line. (This means the student opens the door if it is closed and closes the door if it is open.) Student 4 changes the state of the doors of Lockers 4, 8, 12, 16, and so on. Student 5 changes the state of every fifth door, Student 6 changes the state of every sixth door, and so on, until all 1,000 students have had a turn.

Consider this question:
When all the students have finished, which locker doors are open? Make a conjecture about the answer to this question. Then, describe a strategy you might use to try to find the answer.

Appendix F

Weekly assessments for the two classes.